MAGNETIC FIELD AND HEAT TRANSFER ANALYSIS OF HALBACH MAGNET ARRAYS FOR USE IN MAGNETIC REFRIGERATION SYSTEMS

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ABSTRACT

Due to the entropy change and space limitations in magnetic refrigeration systems, heat transfer enhancement is crucial. Cooling capacity of these systems can be increased by improving magnetic field applied on the magnetocaloric material (MCM) which results in higher entropy change and augmented heat transfer rates. In this study, radial and axial magnetic field variations within the aperture of the magnet array were determined by using the magnetic scalar potential and magnetic field relations through which analytical calculations were performed based on the expansion of the power series and the results were compared to those from the numerical analysis. In both the analytical and numerical analyses, the magnet array was assigned to have same type of permanent magnet. MCM was positioned concentrically inside the bore of the permanent magnet array. Heat transfer fluid flows through the clearance between the magnet array and the MCM. Magnetocaloric effect and corresponding entropy change and behavior of heat transfer rate around the MCM was observed. Results showed that with the cylinder long enough the magnetic field remains constant along the direction of the centerline of the magnet-MCM assembly. Analytical and numerical results were observed to be in close agreement which ensures complex analyses within more elaborate array geometries can be conducted in the future by using this mathematical method developed for domestic magnetic refrigeration systems.

KEYWORDS: Magnetic Refrigeration, Halbach Magnet Array, Magnetization, Entropy Change, Heat Exchange

1. INTRODUCTION

Magnetic refrigeration technology is an innovative alternative to conventional refrigeration systems due to its high theoretical limits and environmentally-friendliness. Alternative refrigeration technologies gained more significance after the environmental protocols such as Kyoto and Montreal examples, and the COP21 meeting in Paris. Besides enabling Freon-free cooling applications, these technologies also have the potential to yield higher COP values resulting in energy savings. According to Coulomb [1], 17% of the global energy consumption is due to HVAC and refrigeration systems. This fact by itself is sufficient to put alternative refrigeration technologies such as magnetic cooling on the showcase because of their promising COP values approaching 15 according to Yu et al. [2]. Magnetic refrigeration has been built on the principle called as magnetocaloric effect which can be explained as an increase at materials temperature with the presence of magnetic field around it. When magnetic field is applied, magnetic dipoles of the material become aligned which causes them to vibrate less and reduce their entropy. According to the second law, as a response the material increases its temperature. When presence of

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magnetic field is removed, material cools down to balance itself out. Rotary and reciprocating systems are the most commonly used ones when it comes to magnetic refrigeration technologies. Numerical, analytical, experimental analysis were conducted to gain an insight and to compare these systems to optimize and come up with the suitable systems for changing conditions. Lozano et al. [3] came up with a numerical model for the rotary type active magnetic refrigerator (AMR) systems. He used Gadolinium (Gd) as magnetocaloric material in a spherical geometry which packed the regenerators of the AMR. 1.24 T is obtained as magnetic flux density with the temperature differences of 16.8 K and 7.1 K for 200 W and 400 W of thermal loads, respectively. Velázquez et al. [4] used permanent type of Halbach magnet array with magnetic field of 1.24 T assigned to model a reciprocating magnetic refrigerator with variable conditions. Gd is used as magnetocaloric material which has radii of 0.1-0.2 mm. Working fluid is employed so that the water and ethylene-glycol ratio is 75% and 25% respectively. As a result, he obtained 20 K to be maximum no-load temperature span and 6W as a cooling capacity when there is no temperature span. Gschneidner et al. [5] conducted a research with different type of magnetocaloric materials and he found out that Gd and Gd-based alloys are a better option when it comes to domestic magnetic refrigeration. Beltran-Lopez et al. [6] studied the alloy group La(Fe1-x-yCoxSiy), and found out that this group is also well-suited for domestic magnetic refrigeration systems. Some studies focus on the magnet orientation side of the overall system, as well. According to the research conducted by Bjørk et al. [7], it is found out that electromagnets yield better results at higher magnetic field values. However, while permanent magnets require no power, power is required to employ electromagnets. He tested the system under variable magnetic field conditions to obtain the relationship. In another research performed by Allah et al. [8], different magnet geometries were employed, and finite element method was used to solve for the magnetic field values. Soltner and Blümber [9] examined permanent magnets which are identically shaped Halbach arrays, also known as magic rings, to build Halbach dipoles. The study also covered flux density of single Halbach arrays and composed 3D geometries. Ravaud and Lemarquand [10] conducted a research about permanent magnets and their magnetic field when they are magnetized uniformly in air. Coulombian model was used to analyze radial, tangential and radial-tangential versions of magnetization direction. As a result of this analytical study magnetic field of the permanent magnets were displayed and it was claimed that this analytical method has low computational cost and is suitable to be used for further studies in the field of Halbach magnet arrays. Camacho and Sosa [11] made an analytical research about magnetic field of a permanent magnet with different geometries. Azimuthal symmetry is assumed for all of the geometries such as sphere, cylinder, cone and so on. Analytical results were done by using Legendre polynomials and were compared with experimental data. It was shown that experimental data harmonizes with analytical model developed.

In this paper, theoretical analysis is compared with the numerical model obtained by using Finite Element Method Magnetics (FEMM) software. NdFEB 52 MGOe type of magnet is used as the permanent magnets and surroundings of the magnet is assumed to be air. Magnetic flux density along radial and axial axes are observed and results of magnetic flux in radial axis are compared.

2. ANALYSIS

2.1 Magnetization Analysis

Numerical analysis. Analyses were conducted by using FEMM software to model the magnet numerically so that the results can be compared with the analytical results. Circular magnet array is defined to the software with an inner and outer radius, \( R_{in} \) and \( R_{out} \), respectively as well as with a length, \( L \), of the magnet. Magnet type and its geometric values can be shown at Table 1.
Table 1 Magnet properties

<table>
<thead>
<tr>
<th>Magnet Type</th>
<th>$R_{ins}$ mm</th>
<th>$R_{out}$ mm</th>
<th>$L$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>NdFeB52 MGOe</td>
<td>40</td>
<td>80</td>
<td>200</td>
</tr>
</tbody>
</table>

The system was designed such that the magnetocaloric material is placed inside the circular Halbach array concentrically with a gap of 5 mm. Halbach array is composed of 24 segments (Fig. 1), with the NdFeB52 MGOe magnets having a remanence magnetic flux density of 1.4 T.

![Circular Halbach magnet array with magnetization direction of each segment](image)

**Fig. 1** Circular Halbach magnet array with magnetization direction of each segment

Magnetization direction of the segments is shown as $\eta$ and it is shown in Fig. 1 with arrows. $\eta$ can be represented by using the angle $\phi$ which is the angle with respect to vertical axis on a specific segment as shown below.

$$\eta = 2\phi$$  \hspace{1cm} (1)

Dirichlet type of boundary is selected and applied to the geometry by using the feature of FEMM. Dirichlet boundary type is used to keep the voltage of a surface in the problem domain which means it focuses the magnetic field and prevents it to dissipate around the geometry. In other words, it prevents magnetic flux to cross the boundary. There are 7 Dirichlet boundaries assigned to the geometry by the software and it is illustrated in Fig. 2.
FEMM treats the problem as a 2D problem and gives the magnetic flux density profile only along X and Y axis and assume magnets are infinitely long so that the effect of the magnetic field along z-direction can be neglected.

The array is assumed to be magnetostatic and the magnetic vector potential and its total value inside the magnet array, bore, for segments of the permanent magnet can be obtained by using the equation below mentioned by Allab et al. [7]:

\[
dA = \frac{1}{4\pi\mu_0} \sum_n \frac{B_r N_n ds_n}{r_n}
\]  

(2)

Here, \(N_n\) is the unit vector normal to the \(n^{th}\) surface element \(ds_n\). \(r_n\) is the distance between the \(n^{th}\) magnet element and a specific point of interest inside the bore. Effect of any NdFeB52 MGOe magnet segment in terms of magnetic field in the circular Halbach array for any given x, y, z coordinate can be obtained by using Eqn. 3.

\[
H = \nabla \times A = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}
\]

(3)
The magnetic field within the bore can be obtained by the superposition of all contributions from the segments of the magnet array as indicated by Eqn. 3. The magnetic flux density, $B$, can be determined from the total magnetic field, $H$, through the known relation,

$$B = \mu_0 H.$$  \hspace{1cm} (4)

where, $\mu_0$ is the permeability of free space.

**Analytical analysis.** In this analytical analysis, the scalar potential approach was used to determine the magnetic flux density at any point inside the bore of a cylindrical Halbach magnet. Circular cross-section was divided into $N$ segments each of which has a constant magnetization with angular width $2(n-1)/N$. The magnetization in Eqn. 5, is consistent with the magnetization direction shown in Fig. 1.

$$M = M_r \left[ \cos \left( \frac{2(n-1)\pi}{N} - \frac{\pi}{2} \right) \rho' + \sin \left( \frac{2(n-1)\pi}{N} - \frac{\pi}{2} \right) \phi' \right]$$  \hspace{1cm} (5)

here, $M_r$ stands for remanent magnetization.

In Fig. 3, The origin is set at the middle of the cylinder. $\rho'$ is the distance from the z-axis to the source volume element, $\rho$ is the distance between the z-axis and the observation point inside the bore, and $R_1$ is the distance between the observation point and the source. $\rho'$, $\rho$, and $R_1$ are in the same plane. In Fig. 4, $\mathbf{r}'$ is the vector position from the origin to the source element and $\mathbf{r}$ is the vector position from the origin to the observation point.

In order to calculate the magnetic flux density, scalar magnetic potential should be known, these two quantities can be related using Maxwell’s equations and given by Eqn. 6:

$$B = -\mu_0 \nabla \Phi_p.$$  \hspace{1cm} (6)

$\Phi_p$, scalar magnetic potential can be computed employing Eqn. 7:

$$d \Phi_p = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|} \frac{M \cdot \mathbf{n}}{|dV|}$$  \hspace{1cm} (7)

where, $dV$ is the source volume element given by $\rho'd\rho'd\phi'dz'$. Furthermore, $\mathbf{n}$ represents the unit vector along the direction of $\mathbf{r} - \mathbf{r}'$ and it is shown in Eqn. 8.
Fig. 3 Geometric parameters used in the mathematical model (2D view)

Fig. 4 Geometric parameters used in the mathematical model (3D view)
In the above equation, the magnitude of $R_1$ can be derived with the aid of Fig. 3:

$$R_1 = \sqrt{\rho'^2 + \rho^2 - 2\rho'\rho\cos(\phi' - \phi)}$$  \(9\)

Then, the potential at the observation point from all segments can be expressed as in Eqn. 10:

$$\Phi_p(z, \rho, \phi) = \frac{M_r}{4\pi} \sum_{n=1}^{N} \int_{(n-1)\frac{2\pi}{N}}^{(n)\frac{2\pi}{N}} \int_{R_{in}}^{R_{out}} \frac{\beta(z - L/2)}{R_1^2 \sqrt{R_1^2 + (z - L/2)^2}} \rho' dp' d\phi' - \frac{\beta(z + L/2)}{R_1^2 \sqrt{R_1^2 + (z + L/2)^2}} \rho dp \sin \left( \frac{(n-1)\pi}{N} - \frac{\pi}{2} \right)$$  \(10\)

where $R_{in}$ is the inner radius of the magnet, and $R_{out}$ is the outer radius of the magnet. The parameter $\beta$ is defined below in Eqn. 11.

$$\beta = \beta_\rho \cos \left( \frac{2(n-1)\pi}{N} - \frac{\pi}{2} \right) + \beta_\phi \sin \left( \frac{2(n-1)\pi}{N} - \frac{\pi}{2} \right)$$  \(11\)

The parameters $\beta_\rho$ and $\beta_\phi$ in Eqn. 11 are defined by the following relations.

$$\beta_\rho = \cos(\phi') [\rho'\cos(\phi') - \rho\cos(\phi)] + \sin(\phi') [\rho'\sin(\phi') - \rho\sin(\phi)]$$  \(12\)

$$\beta_\phi = \cos(\phi') [\rho'\sin(\phi') - \rho\sin(\phi)] - \sin(\phi') [\rho'\cos(\phi') - \rho\cos(\phi)]$$  \(13\)

### 2.2 Heat Transfer Analysis

After magnetic field calculations are done by using numerical and analytical methods, the corresponding magnetic flux density, $B$, can be found and used to determine the heat flux of the MCM dispersed onto the working fluid. The working fluid is selected as water whereas the MCM is Gadolinium. Their properties are shown in Table 2.

<table>
<thead>
<tr>
<th>Type</th>
<th>Density, $\rho$, kg/m$^3$</th>
<th>Thermal Conductivity, $k$, W/mK</th>
<th>Specific Heat, $c_p$, J/kgK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gadolinium</td>
<td>7901</td>
<td>11</td>
<td>240</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>0.6</td>
<td>4184</td>
</tr>
</tbody>
</table>
In order to find the heat flux, we first find the corresponding entropy span following the work of Yu et al. [2] where the corresponding entropy change of Gadolinium under different magnetic flux density values are observed. Then, Eqn. 14 referenced from the work of Allab et al. [7], is used to calculate the total energy.

\[
Q_c = \rho_{\text{MCM}} V_{\text{MCM}} \Delta S T_c
\]  

(14)

In Eqn. 14, \(\rho_{\text{MCM}}\) and \(V_{\text{MCM}}\) stand for density and the volume of the magnetocaloric material and \(\Delta S\) and \(T_c\) represent the entropy change and the cold water temperature, respectively. With the total energy of Gd calculated, energy harvest of water from Gd can be found by using the governing equations. Eqn. 15, 16 and 17 represent the simplified version of continuity, momentum, and the energy equations, respectively.

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = -\frac{\partial \rho}{\partial t}
\]  

(15)

\[
v_z(r) = \frac{1}{4\mu} \frac{dp}{dz} \left[ r^2 - r_i^2 \right] - \left( r_o^2 - r_i^2 \right) \frac{\ln(r/r_i)}{\ln(r_o/r_i)}
\]  

(16)

\[
\rho c_p \left( v_z \frac{\partial T}{\partial z} \right) = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \mu \left( \frac{1}{4\mu} \frac{dp}{dz} \left[ 2r - \frac{r_o^2 - r_i^2}{r \ln(r_o/r_i)} \right] \right)^2
\]  

(17)

ANSYS was used to obtain the temperature distribution over the fluid domain. Total energy, wet surface area, \(A_w\), of the MCM are known and the magnetization time, \(t\), is selected as 10 seconds. Mass flow rate of the working fluid, water, is 0.03 kg/s. Initial temperature of water is 20°C. Fig. 5 represents the meshing done using the software. Mapped face meshing and face sizing methods are used to ensure the quality of the mesh. Furthermore, 5 layers are given to the thickness of the volume to observe temperature change radially.
3. RESULTS

Results obtained from the FEMM showed that magnetic field is nearly uniform along the bore and the change only occurs at the areas very close to the edges. Average magnetic flux density is around 0.953 Tesla. Figs. 6 and 7 illustrate the magnetic flux density contour and magnetic flux density inside the bore, respectively.

**Fig. 6** Magnetic flux density contour.

**Fig. 7** Magnetic flux density inside the bore.
As a result of analytical calculations, it is found that the magnetic field is in the \( y \)-direction, consistent with the software results, and very much uniform inside the bore at any given \( z \). However, due to edge effects the field strength varies slightly along the \( z \) direction as can be seen in Figs. 8 and 9.

**Fig. 8** Magnetic flux density as a function of axial distance

Magnetic flux density as a function of axial distance within the bore shown in Fig. 8 is obtained using \( B_r = 1.4 \text{T}, \rho = 3.5 \text{cm}, \phi = 0 \) and the values in Fig. 9 are obtained using \( B_r = 1.4 \text{T}, z = 0, \phi = 0 \).
As a result of the heat transfer analysis, corresponding Reynolds number, entropy change and heat flux from the Gd to the water are listed in Table 3.

### Table 3 Results obtained for the flow

<table>
<thead>
<tr>
<th>Cross-sectional area, $A_c$, m$^2$</th>
<th>0.00118</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number, Re</td>
<td>254.65</td>
</tr>
<tr>
<td>Entropy change, $\Delta S$, J/kgK</td>
<td>0.953</td>
</tr>
<tr>
<td>Heat flux, $q$, W/m$^2$</td>
<td>9973.5</td>
</tr>
</tbody>
</table>

The obtained heat flux value is used to define the boundary between the MCM and water. Temperature distribution for the working fluid is illustrated in Fig. 10. Radial and axial temperature distribution of the water at certain Z and R values respectively are given in Figs. 11 and 12. Temperature span and capacity values calculated are listed in Table 4.

![Temperature distribution](image)

**Fig. 10** Temperature distribution of the working fluid within the aperture
**Fig. 11** Radial temperature distribution of the working fluid.

**Fig. 12** Axial temperature distribution of the working fluid.
4. CONCLUSION

A circular Halbach magnet array with a circular magnetocaloric material positioned concentrically at its center were examined in terms of local magnetic field values and corresponding entropy and temperature changes. The study was conducted on predefined geometric properties and numerical results were achieved. It was found out that the mathematical model built to calculate the field was in agreement with the results from numerical analysis which is promising for further studies where more elaborate geometries such as polygonal magnet arrays are examined. It was observed that as the length of the magnet increases, edge effects in terms of magnetization become negligible. In addition, the analysis revealed that the length of the magnet slightly affects the magnetic flux density radially. It was detected that the temperature of the working fluid increases in both radial and axial directions, as expected. With the selected capacities of magnets and size of magnetocaloric material, a 4.5 K temperature span was achieved, which is promising for the selected size of the assembly. Further analysis on geometric optimization including different cylindrical dimensions and varied geometric shapes such as polygonal magnet-MCM assemblies would be beneficial to advance this study, and are planned to be conducted as future work.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Property</th>
<th>Circular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature span, K</td>
<td>4.5</td>
</tr>
<tr>
<td>Capacity, $q_c$, W</td>
<td>564.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>(m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$ Cross-sectional area of the magnetocaloric material</td>
<td></td>
</tr>
<tr>
<td>$A_{cw}$ Cross-sectional area of the aperture between the magnet and MCM</td>
<td></td>
</tr>
<tr>
<td>$A_s$ Wet surface area of the magnetocaloric material</td>
<td></td>
</tr>
<tr>
<td>$A_x$ Magnetic vector potential in the direction of x</td>
<td>(Vs/m)</td>
</tr>
<tr>
<td>$A_y$ Magnetic vector potential in the direction of y</td>
<td>(Vs/m)</td>
</tr>
<tr>
<td>$A_z$ Magnetic vector potential in the direction of z</td>
<td>(Vs/m)</td>
</tr>
<tr>
<td>$B$ Magnetic flux density</td>
<td>(T)</td>
</tr>
<tr>
<td>$B_r$ Remanence magnetic flux density</td>
<td>(T)</td>
</tr>
<tr>
<td>$dV$ The source volume element</td>
<td>(m$^3$)</td>
</tr>
<tr>
<td>$H$ Total magnetic field</td>
<td>(A/m)</td>
</tr>
<tr>
<td>$k$ Thermal conductivity</td>
<td>(W/mK)</td>
</tr>
<tr>
<td>$L$ Length of the magnet</td>
<td>(m)</td>
</tr>
<tr>
<td>$M$ Magnetization</td>
<td>(A/m)</td>
</tr>
<tr>
<td>$M_r$ Magnitude of magnetization</td>
<td>(-)</td>
</tr>
<tr>
<td>$m$ Mass flow rate of the water</td>
<td>(kg/s)</td>
</tr>
<tr>
<td>$N_n$ Unit vector normal to the $n^{th}$ surface element $d_{Sn}$</td>
<td>(-)</td>
</tr>
<tr>
<td>$n$ The $n^{th}$ segment</td>
<td>(-)</td>
</tr>
<tr>
<td>$Q_c$ Total energy</td>
<td>(J)</td>
</tr>
<tr>
<td>$q$ Heat flux</td>
<td>(W/m$^2$)</td>
</tr>
<tr>
<td>$q_c$ Capacity</td>
<td>(W)</td>
</tr>
</tbody>
</table>
The distance of the observation point and the source element \( R_1 \) (m)

The vector portion from the origin to the observation point \( r \) (m)

The vector position from the origin to the source element \( r' \) (m)

Inner radius \( R_{in} \) (m)

Outer radius \( R_{out} \) (m)

Distance between the \( n^{th} \) magnet element and a specific point inside the bore \( r_n \) (m)

Amount of time of the magnetization process \( t \) (s)

Cold water temperature \( T_c \) (K)

Volume of magnetocaloric material \( V_{MCM} \) (m³)

Velocity of the water \( V_w \) (m/s)

Distance from center to the observation point \( z \) (m)

Distance from center to the source element \( z' \) (m)

Magnetization direction of the segments \( \eta \) (rad)

Azimuthal angle to the observation point \( \phi \) (rad)

Azimuthal angle to the source element \( \phi' \) (rad)

Scalar magnetic potential at observation point \( \Phi_p \) (A)

Permeability of free space \( \mu_0 \) (N/A²)

Dynamic viscosity \( \mu \) (J/kgK)

The distance from the z-axis to the source magnetic moment \( \rho' \) (m)

The distance between the z-axis and the observation point inside the bore \( \rho \) (m)

Density of magnetocaloric material \( \rho_{MCM} \) (kg/m³)

Entropy change of the magnetocaloric material \( \Delta S \) (J/kgK)

Radius of magnetocaloric material \( R_{MCM} \) (m)

REFERENCES


