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A NUMERICAL STUDY OF ASSISTING MIXED CONVECTIVE HEAT TRANSFER FROM NARROW ISOTHERMAL VERTICAL FLAT PLATES

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ABSTRACT

Many studies of mixed convective heat transfer from heated flat plates with the forced flow parallel to the plate surface are available, most of these studies dealing with wide plates. However, practical situations exist in which there is effectively mixed convective heat transfer from narrow plates and in such cases the relative plate width is expected to have a significant influence on the heat transfer rate. The present study numerically investigates how the relative width of the plate affects the mixed convective heat transfer rate from a thin narrow vertical plate. It has been assumed that the plate surfaces are isothermal. The Boussinesq approach has been adopted. The solution has been obtained using ANSYS FLUENT©. The mean heat transfer rate from the heated surface of the plate has been expressed in terms of a Nusselt number based on the length of the plate, this Nusselt numbers being dependent on the Rayleigh number and the Reynolds number based on the plate length, on the ratio of the plate width to the plate length, and on the Prandtl number. Results have only been obtained for a Prandtl number of 0.74. Variations of the Nusselt numbers with Rayleigh number and with Reynolds number for various dimensionless plate widths have been obtained, the results showing that the effect of the plate width on the heat transfer rate can be significant.

KEY WORDS: Mixed convection, Combined convection, Vertical plate, Narrow plate, Numerical

1. INTRODUCTION

In some situations in which there is a forced flow over a body and in which temperature differences exist in the flow, the buoyancy forces in the flow that arise due to the density differences resulting from the temperature differences in the flow have a significant effect on the flow and the heat transfer despite the presence of the forced flow. Such flows are termed mixed or combined natural and forced convective flows. There have been a number of studies of mixed convective heat transfer from vertical flat plates. Typical of these studies are those described in [1-12]. Some studies of mixed convective heat transfer from horizontal plates, [13-15], and from inclined plates, [16-20], have also been undertaken. Essentially all of the studies of mixed convective heat transfer from vertical flat plates mentioned above have been concerned with wide plates, i.e., with situations in which the flow over the plate can be assumed to be two-dimensional. However, there are a number of practical situations that effectively involve heat transfer from narrow plates, i.e., from plates whose width is relatively small compared to their length. In such cases a three-dimensional flow exists near the edges of the plate and in particular near the edges of the plate which are parallel to the direction of the forced flow over the plate. There have been some studies of natural convective heat transfer from narrow plates. A review of many of these studies, which show that with natural convection the edge effect tends to increase the heat transfer rate, is given in [21].

The magnitude of the edge effect on the heat transfer rate in mixed convective flow will depend on the relative width of the plate compared to its length in the flow direction and on whether the forced flow is in the same direction as the buoyancy forces (assisting flow) or is in the opposite direction to the buoyancy

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forced (opposing flow). Attention has here been limited to assisting flow. In order to obtain an indication of the importance of this edge effect on the heat transfer rate, attention has here been given to heat transfer from a vertical, narrow, thin, isothermal flat plate placed in a vertical forced fluid flow; the flow situation considered is therefore as shown in Fig. 1. In the present study the effect of the buoyancy forces on the flow and therefore on the heat transfer rate has been studied assuming that the flow remains laminar.

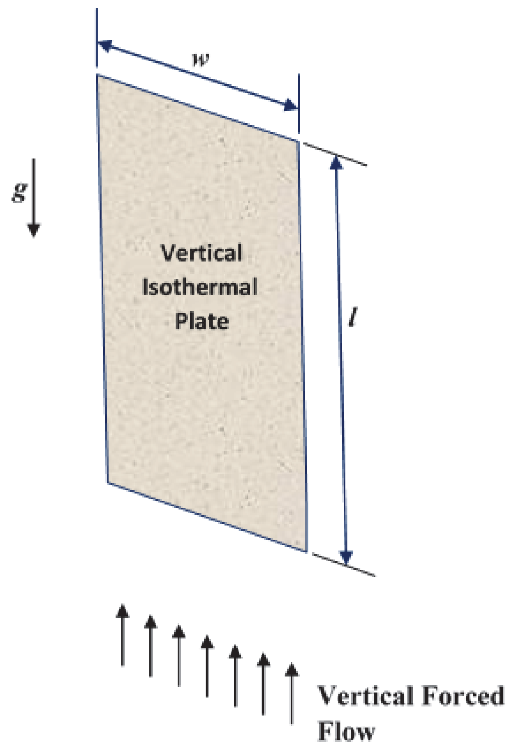

Fig. 1 Flow situation considered.

2. SOLUTION PROCEDURE

The flow, which was assumed to remain laminar, has been assumed to be steady and to be symmetrical about the center-plane through the plate. The Boussinesq approach has been adopted in dealing with the buoyancy forces, i.e., basically it was assumed that the density changes that give rise to the buoyancy forces are proportional to the difference between the local temperature in the fluid and the temperature of the fluid far from the plate and that the effects of all other fluid property changes resulting from the temperature changes in the flow were negligible. The solution has been obtained using the commercial CFD solver ANSYS FLUENT©. Extensive grid- and convergence criterion independence testing was undertaken. This testing indicated that the mean heat transfer results presented here are to within 1% independent of the number of grid points and of the convergence-criterion used.

3. RESULTS

The mean heat transfer rate from the surface of the plate have been expressed in terms of a mean Nusselt number based on the plate length, *l,* i.e., the following Nusselt number has been used:

$$Nu = \frac{q'_{mean}\, l}{\left(T_w - T_f\right)k} \tag{1}$$

where T_w and T_f are the temperatures of the isothermal plate surfaces and of the undisturbed fluid far from the plate, respectively. This Nusselt number will depend on the following:

1. The Rayleigh number, *Ra*, based on the plate length, *l*, i.e.:

$$Ra = \frac{\beta g (T_w - T_f) l^3}{\nu^2} Pr \tag{2}$$

2. The Reynolds number, *Re*, based on the plate length, *l*, and on the forced velocity in the undisturbed flow ahead of the plate, *U:*

$$Re = \frac{U\, l}{\nu} \tag{3}$$

3. The relative width of the plate, *W*:

$$W = \frac{w}{l} \tag{4}$$

4. The Prandtl number, *Pr* .

Results have only here been obtained for Pr = 0.7, i.e., essentially the value for air at ambient conditions. Hence:

$$Nu = \text{function}\left(Ra, Re, W\right) \tag{5}$$

Typical variations of *Nu* with *Ra* for various values of *W* are shown in Figs. 2, 3, 4, and 5, each of these figures giving results for a different value of *Re*. It will be seen from the results given in these figures that, under all situations considered, when *W* decreases there is an increase in the heat transfer rate under all conditions considered. It will also be seen from Figs. 4 and 5 that at the higher values of *Re* considered the value of *Nu* for a given value of *W* becomes constant at the lower values of *Ra* considered, i.e., purely forced convective flow exists under these conditions. It will also be noted from these results that the value of *Ra* at which the flow becomes essentially purely forced convective at a given value of *Re* is only weakly dependent on the value of *W*.

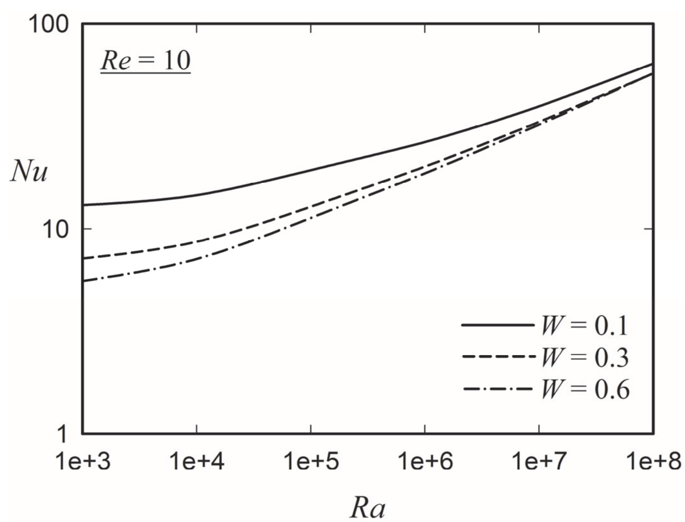

Fig. 2 Variation of Nusselt number with Rayleigh number for various dimensionless plate widths for a Reynolds number of 10.

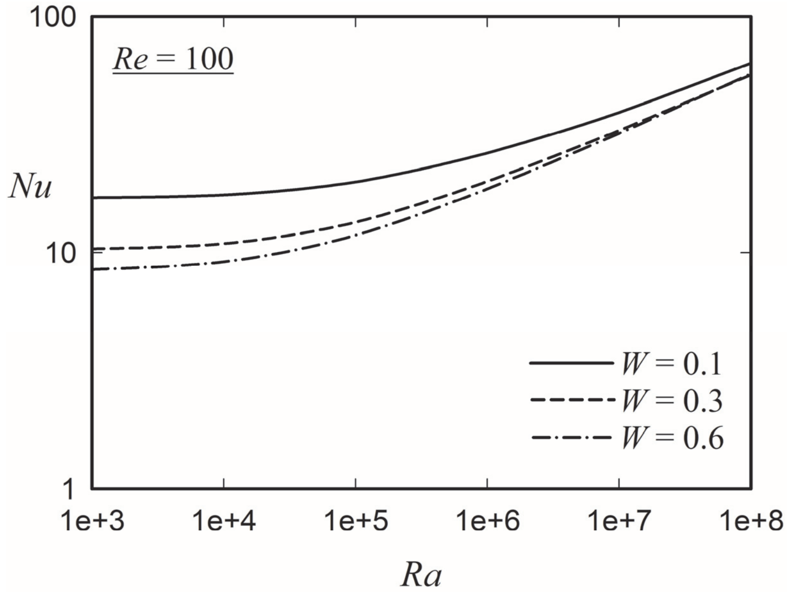

Fig. 3 Variation of Nusselt number with Rayleigh number for various dimensionless plate widths for a Reynolds number of 100.

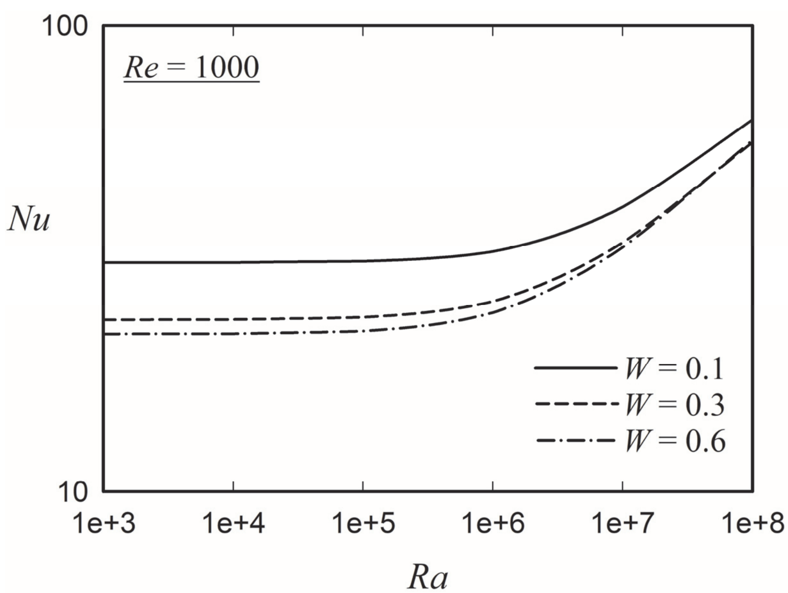

Fig. 4 Variation of Nusselt number with Rayleigh number for various dimensionless plate widths for a Reynolds number of 1000.

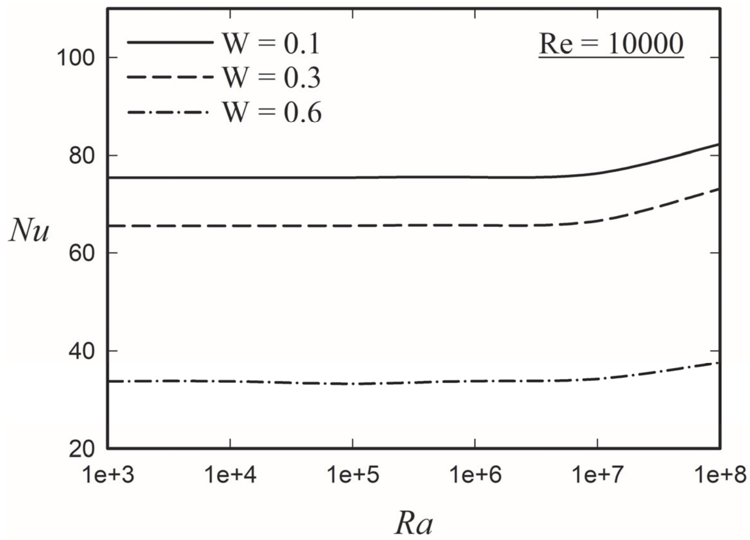

Fig. 5 Variation of Nusselt number with Rayleigh number for various dimensionless plate widths for a Reynolds number of 10,000.

To illustrate the conditions under which purely natural convective flow exists attention will next be given to the variation of *Nu* with *Re*. Typical variations of *Nu* with *Re* for various values of *W* are shown in Figs. 6, 7, 8, and 9, each of these figures giving results for a different value of *Ra*. It will again be seen from the results given in these figures that, for all situations considered, when *W* decreases there is an increase in the heat transfer rate.

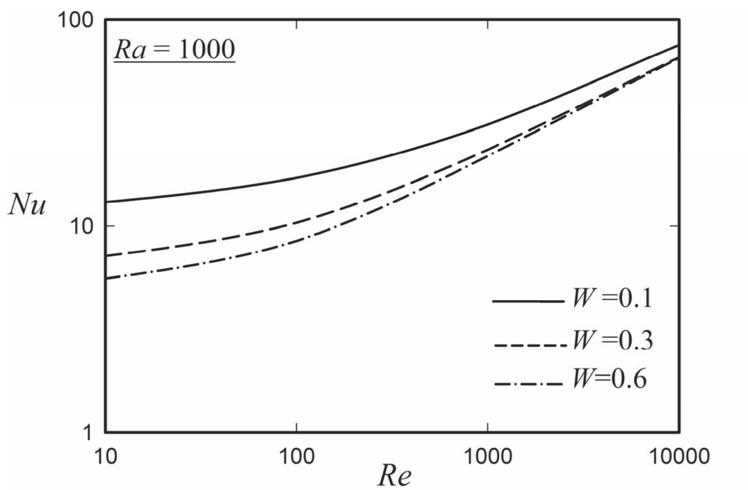

Fig. 6 Variation of Nusselt number with Reynolds number for various dimensionless plate widths for a Rayleigh number of 1000.

It will also be seen from Figs. 8 and 9 that at the higher values of *Ra* considered the value of *Nu* for a given value of *W* becomes constant at the lower values of *Re* considered, i.e., purely natural convective flow exists under these conditions. It will also be noted from these results that the value of *Re* at which the flow becomes essentially purely natural convective at a given value of *Ra* is only weakly dependent on the value of *W*.

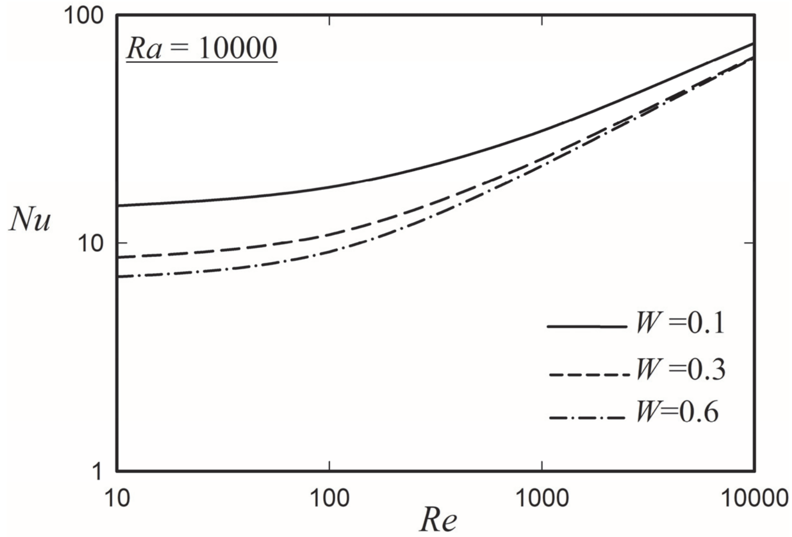

Fig. 7 Variation of Nusselt number with Reynolds number for various dimensionless plate widths for a Rayleigh number of 10,000.

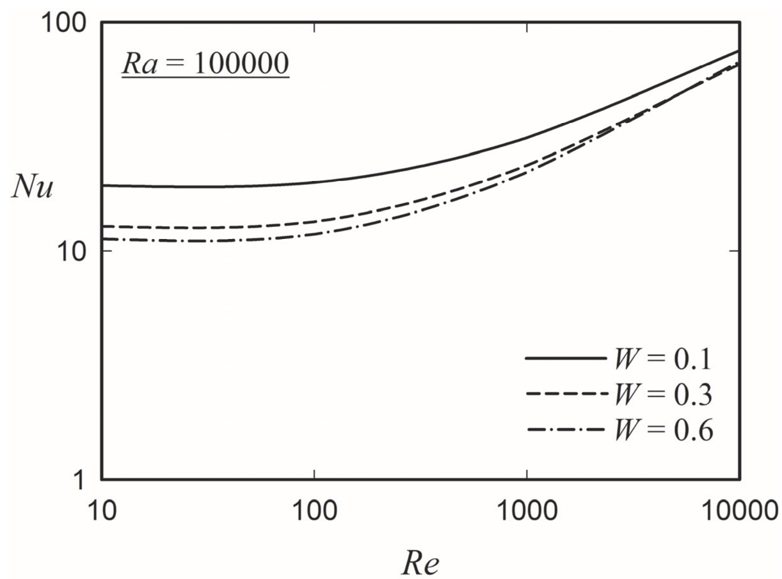

Fig. 8 Variation of Nusselt number with Reynolds number for various dimensionless plate widths for a Rayleigh number of 100,000.

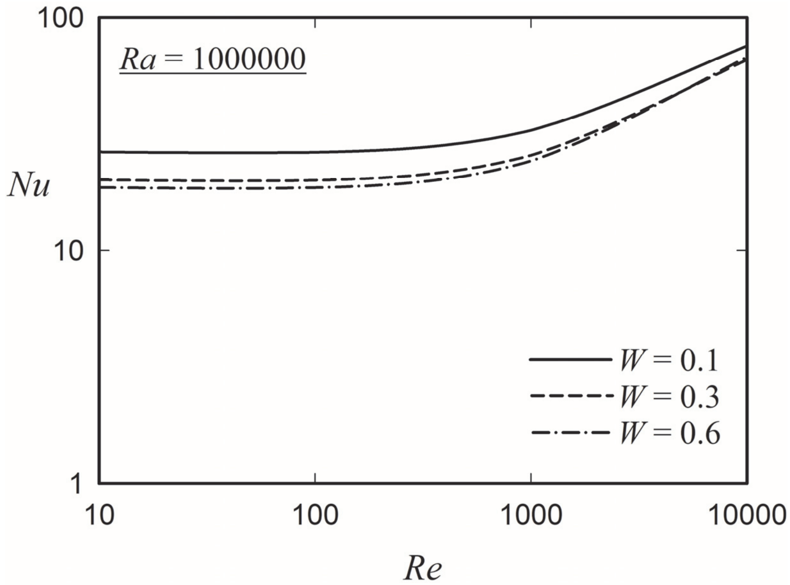

Fig. 9 Variation of Nusselt number with Reynolds number for various dimensionless plate widths for a Rayleigh number of 1,000,000.

To more clearly illustrate the effect of W on the heat transfer rate, typical variations of Nu with W for various values of Ra are shown in Figs. 10, 11, 12, and 13, each of these figures giving results for a different value of Re. These figures show that under all conditions considered Nu increases as the relative width of the plate, W, decreases.

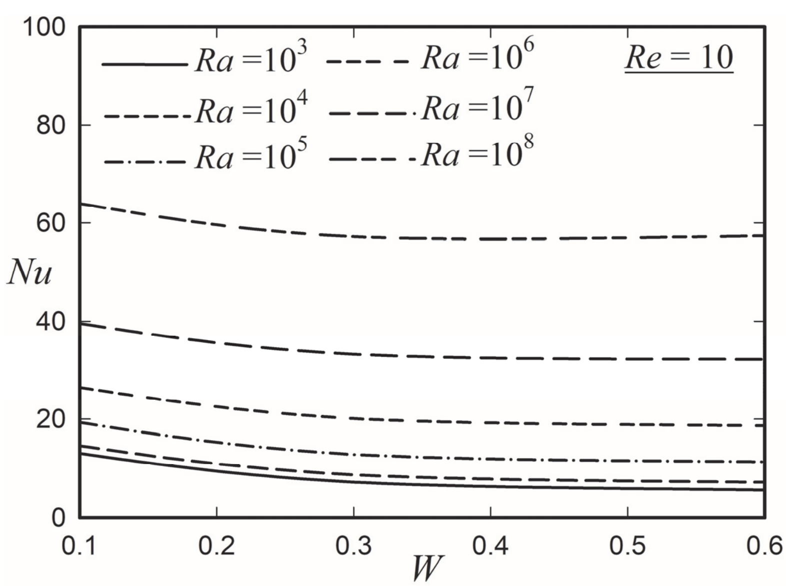

Fig. 10 Variation of Nusselt number with dimensionless plate width for various Rayleigh numbers for a Reynolds number of 10.

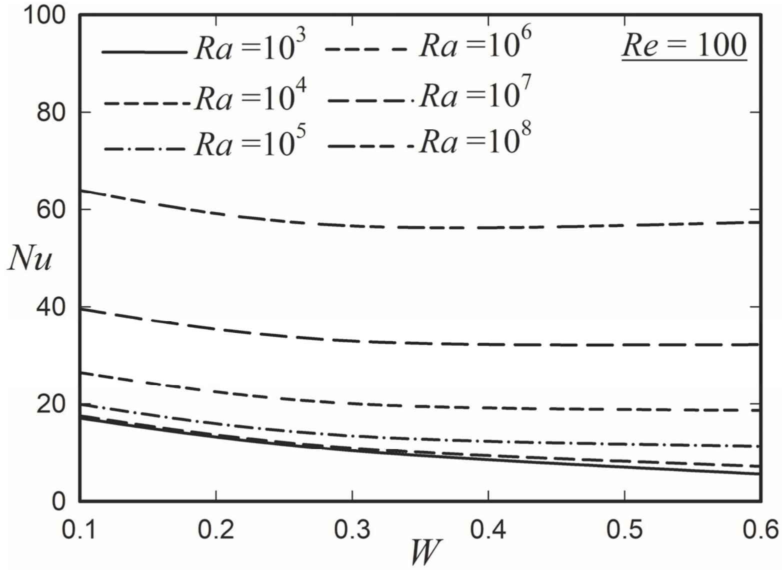

Fig. 11 Variation of Nusselt number with dimensionless plate width for various Rayleigh numbers for a Reynolds number of 100.

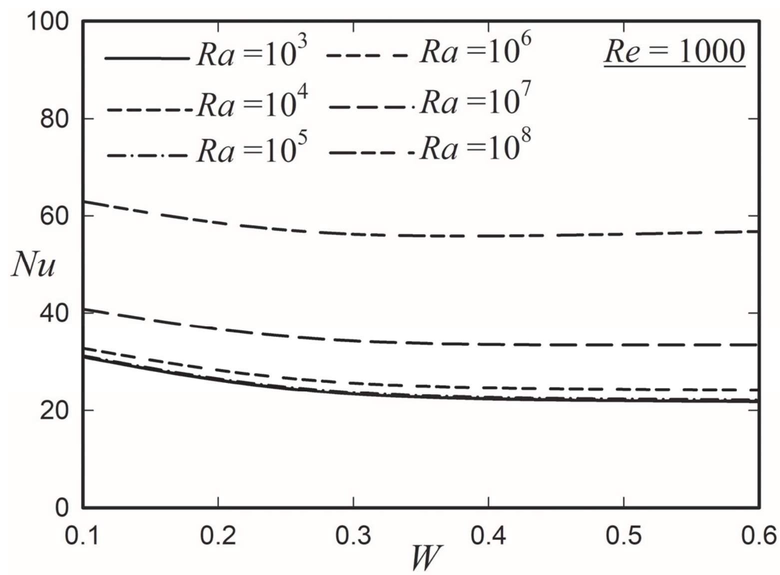

Fig. 12 Variation of Nusselt number with dimensionless plate width for various Rayleigh numbers for a Reynolds number of 1000.

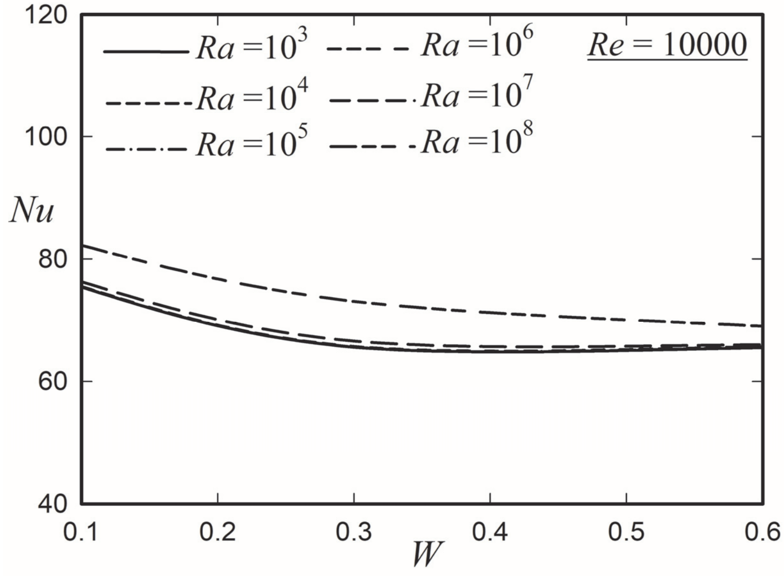

Fig. 13 Variation of Nusselt number with dimensionless plate width for various Rayleigh numbers for a Reynolds number of 10,000.

As mentioned earlier, the results discussed above indicate that the conditions under which the flow becomes essentially purely forced convective or under which the flow becomes essentially purely natural convective are very weakly dependent on the dimensionless plate width, *W*. The conditions under which the flow becomes essentially purely forced convective or under which the flow becomes essentially purely natural convective are, when the effects of *W* are ignored, as shown in Figs 14 and 15.

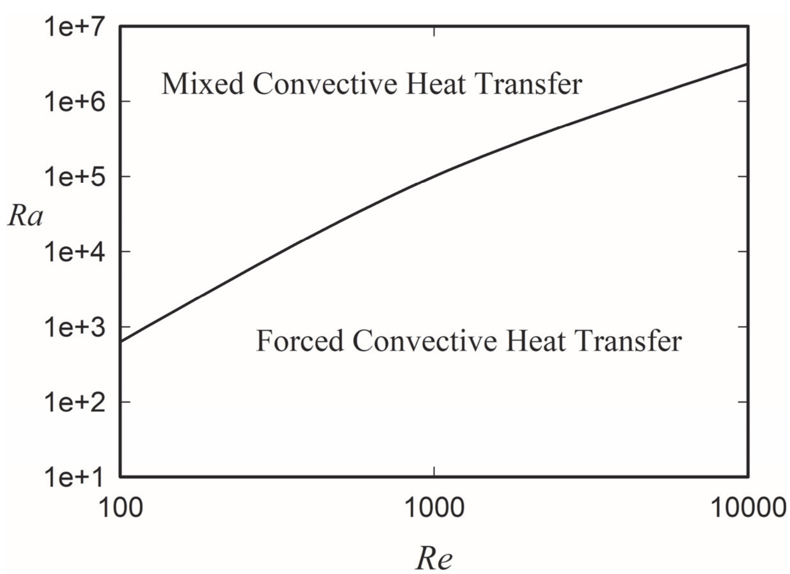

Fig. 14 Conditions under which purely forced convective flow occurred with the conditions considered in the present study.

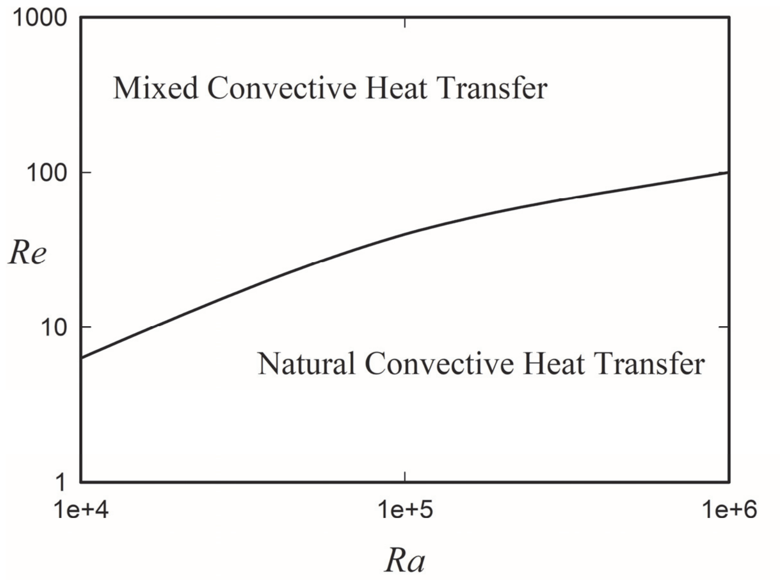

Fig. 15 Conditions under which purely natural convective flow occurred with the conditions considered in the present study.

4. CONCLUSIONS

The results of the present study clearly show that with mixed convective heat transfer from narrow vertical plates the relative width of the plate does have a very significant influence on the heat transfer rate from the plate, the heat transfer rate from the plate increasing as the relative width of the plate, *W*, decreases. For the conditions considered in the present study, the effect of the relative width of the plate on the heat transfer rate was found to be significant for values of *W* less than approximately 0.6.

ACKNOWLEDGEMENT

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NOMENCLATURE

g	gravitation acceleration	(m/s^2)
k	thermal conductivity	
l	length of heated plate	(m)
Nu	mean Nusselt number	(-)
Pr	Prandtl number	(-)
q'_{mean}	mean entire surface heat flux per unit surface area	(W/m^2)
Ra	Rayleigh number	(-)
Re	Reynolds number	(-)
T_w	uniform plate wall temperature	(^{o}K)
T_f	fluid temperature in undisturbed fluid	(^{o}K)
U	forced velocity	(m/s)
W	relative width of plate (w/l)	(-)
w	width of plate	(m)
β	thermal expansion coefficient	(1/K)
ν	kinematic viscosity	(m^{2}s)

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